ON ANALYSIS OF ELECTROMAGNETIC FIELDS USING DEFLATION METHODS

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Poor matrix conditioning often gives rise to severe problems in finite element (FE) analysis of electromagnetic fields. For example, one has poor conditioning when the element shapes are quite flat or distorted. Moreover, one encounters such a problem when the operating frequency or conductivity is set to small in the eddy current analysis based on the A method whose unknown variable is the vector potential.

The matrix deflation, which replaces small eigenvalues with zeros to improve matrix conditioning, is expected to cure the above mentioned problems. The deflation technique has been applied to the conjugate gradient (CG) method with application to boundary value problems [1], and has also applied to diffusion problems for soil layers with significantly different permeability [2]. Moreover, the deflated CG method has been introduced on the basis of the Lanczos algorithm [3]. In computational electromagnetism, magnetostatic problems with large jumps in the magnetic permeability have been analyzed using the deflation method where the approximated eigenvectors corresponding to small eigenvalues are obtained from simplified FE models with local supports [4].

Besides the matrix deflation, the explicit error correction (EEC) and implicit error correction (IEC) techniques have been introduced to eliminate the problematic slowly converging modes [5]. These methods can be recognized as a generalization of the multigrid method and are also relevant to the AV method which employs an augmented FE matrix. Moreover, convergence of the CG method applied to FE analysis of magnetostatic problems with considerably flat elements has been shown to be drastically improved by the singularity decomposition (SD) [6], which has the same basis as the IEC. However, the reason why these methods can improve the matrix conditioning has been unclear.

In this report, the mathematical properties of the above mentioned methods are discussed from a view point of the matrix deflation. In particular, the property of the augmented matrix used in the IEC-SD and AV method is analyzed in detail. It is concluded that the augmented matrices have good conditioning after the preconditioning. Numerical results are shown to support this result.

References

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